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عنـــــــوان

معادلات دیفرانسیل

Subject :

Year: Month. Date.

ABADANOMRAN

معادله همگن زیر را حل کنید. 1

$(5x-2y)y' - y = 2x$

$y' = \frac{y+2x}{5x-2y}$

$z'x + z = \frac{2x+2y}{5x-2zx} = \frac{x(z+2)}{x(5-2z)}$

$z'x = \frac{z+2}{5-2z} - \frac{z}{1} = \frac{z+2-5z+2z^2}{5-2z} = \frac{2z^2-4z+2}{5-2z}$

$\frac{dz}{dx}x = \frac{2z^2-4z+2}{5-2z} \Rightarrow \int \frac{5-2z}{2z^2-4z+2}dz = \int \frac{dx}{x}$

$-\frac{1}{2}\int \frac{(2z-2)-3}{(z^2-2z+1)}dz$

$-\frac{1}{2}\left[\int \frac{2z-2}{z^2-2z+1}dz - \int \frac{3}{z^2-2z+1}dz\right]$

$(z-1)^2$

$-\frac{1}{2}\left[\ln(z^2-2z+1) + \frac{3}{z-1}\right] = \ln x + C$

معادله دیفرانسیل خطی مرتبه اول

$$y' + y\cot g\alpha = \frac{1}{\sin\alpha}$$

$$y = e^{-\int P(\alpha)d\alpha}\left(\int q(\alpha)e^{\int P(\alpha)d\alpha}d\alpha + C\right)$$

$$y = e^{-\int \cot g\alpha}\left(\int \frac{1}{\sin\alpha}e^{\int \cot g\alpha\, d\alpha}d\alpha + C\right)$$

$$y = e^{-\ln\sin\alpha}\left(\int \frac{1}{\sin\alpha}e^{\ln\sin\alpha} + C\right) \Rightarrow$$

$$y = \frac{1}{\sin\alpha}\left(\int 1\,d\alpha + C\right) \Rightarrow y = \frac{1}{\sin\alpha}\left(\alpha + C\right)$$

یکی از دو معادله زیر را حل کنید. برنولی

$$\begin{cases} xy' + \frac{y}{2\ln x} = y^2 \\ y' + \frac{1}{x}y = -2xy^2 \end{cases}$$

$$xy' + \frac{y}{2\ln x} = y^2$$

$$y' + \frac{1}{2x\ln x}y = \frac{1}{x}y^2 \rightarrow y^{-2}y' + y^{-2}\frac{1}{2x\ln x}y = y^{-2}\frac{1}{x}y^2$$

$$z = y^{1-2} \Rightarrow z = y^{-1} \Rightarrow z' = \boxed{-2}\,y^{-3}y'$$

$$z' - \frac{1}{2x\ln x}z = -\frac{1}{x}$$

$$z = e^{-\int \frac{-1}{2x\ln x}dx}\left(-\frac{1}{x}e^{\int -\frac{1}{2x\ln x}dx} + c\right) \qquad u = \ln x$$

$$z = e^{\frac{1}{2}\ln\ln x}\left(-\frac{1}{x}e^{-\frac{1}{2}\ln\ln x}dx + c\right)$$

$$z = (\ln x)^{\frac{1}{2}}\left(\int -\frac{1}{x}\frac{1}{(\ln x)^{\frac{1}{2}}}dx + C\right) \Rightarrow -\int \frac{1}{u^{\frac{1}{2}}} = -2\sqrt{u}$$

$y^{-1} = z_1 (\ln x)^{\frac{1}{2}} \left(-2\sqrt{\ln x + C}\right)$ (1)

$y'' - y' - 2y = 2 e^{4x}$ اولین ضرایب ثابت

$k^2 - k - 2 = 0 \Rightarrow (k-2)(k+1) = 0 \begin{cases} k=2 & y_1 = e^{2x} \\ k=-1 & y_2 = e^{-x} \end{cases}$

جواب عمومی معادله $\Rightarrow y = C_1 e^{2x} + C_2 e^{-x}$

حدس $y = Ae^{4x}$ حدس درست است

$y' = 4Ae^{4x} \Rightarrow 16A - 4A - 2A = 20 \rightarrow 10A = 20 \Rightarrow \boxed{A=2}$

$y'' = 16Ae^{4x}$

جواب خصوصی معادله $= y = C_1 e^{2x} + C_2 e^{-x} + 2e^{4x}$

$y'' - y' = 2n - 1$ (5)

حدس $y = An + B \Rightarrow y = An^2 + Bn \Rightarrow y' = 2An + B \Rightarrow y'' = 2A$

حدس غلط بود.

معادله مشخصه $\Rightarrow k^2 - k = 0$

در معادله

$-2A = 2 \Rightarrow A = -1$

$2A - B = -1 \quad -2 - B = -1 \quad B = -1$

جواب خصوصی $y = -n^2 - n$

$n^2 y'' + 4ny' - 4y = 0$ تعیین جواب انتزوی جواب دیگر

$y_1 = n \quad y_2 = v y_1 \Rightarrow y_2 = \frac{n}{-5}$ -4

$$v = \int \frac{1}{y_1^2} e^{-\int P(n)\,dn}\,dn$$

$$v = \int \frac{1}{n^2} e^{-\int \frac{4}{n}\,dn}\,dn = \int \frac{1}{n^2} e^{-4\ln n}\,dn$$

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$$= \int \frac{1}{x^2}\,\frac{1}{x^4}\,dx = \int x^{-6}\,dx = \frac{x^{-5}}{-5}$$ 6

$$\begin{cases} y'' + y = \sec x \\ y'' + y = \tan x \end{cases}$$

$$k^2 + 1 = 0$$

$$k = \pm i \begin{cases} \alpha = 0 \\ \beta = 1 \end{cases} \begin{cases} y_1 = \sin x \\ y_2 = \cos x \end{cases}$$

$$W = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -1$$

$$v_1 = \int \frac{-\cos x \tan x}{-1}\,dx = -\cos x$$

$$v_2 \int \frac{\sin x \tan x}{-1}\,dx = -\int \frac{\sin^2 x}{\cos x}\,dx = -\int \frac{1-\cos^2 x}{\cos x}\,dx$$

$$-\int \sec x - \cos x\,dx = -\left[\ln(\sec x + \tan x) - \sin x\right]$$

جواب عمومی $\Rightarrow y = v_1 y_1 + v_2 y_2$

7. تعیین جواب $x^2y'' - 2xy' + 2y = 4\ln x$

$z = \ln x$ $\quad y'' - 3y' + 2y = 4e^{2z}$

$k^2 - 3k + 2 = 0 \rightarrow (k-2)(k-1) = 0$ $\quad \begin{cases} k=1 & y_1 = e^z \\ k=2 & y_2 = e^{2z} \end{cases}$

جواب عمومی معادله $y_c = C_1 e^z + C_2 e^{2z}$

جواب خصوصی $y = Ae^{2z}$ حدس مناسب توابع پایه

$y = Aze^{2z}$

$y' = Ae^{2z} + 2Aze^{2z}$ جواب خصوصی $\quad 4A - 3A = 4 \Rightarrow A = 4$

$y'' = 2Ae^{2z} + 2Ae^{2z} + 4Aze^{2z}$

جواب خصوصی $y = 4ze^{2z}$

$$L\left[\int_0^x e^x \cos 2x\,dx\right] = \frac{\frac{p-1}{(p-1)^2+4}}{p} \quad .8$$

$$\frac{p}{p^2+4}$$

$$L^{-1}\left[\frac{1}{p(p^2+1)}\right] = L^{-1}\left[\frac{\frac{1}{p^2+1}}{p}\right] \Rightarrow \int_0^x \sin x\,dx$$

$$L^{-1}\left[\frac{p}{p^2-2p+26}\right] = L^{-1}\left[\frac{p-1+1}{(p-1)^2+25}\right] = L^{-1}\left[\frac{(p-1)}{(p-1)^2+25} + \frac{1}{(p-1)^2+25}\right]$$

$$= e^x \cos 5x + e^x \frac{1}{5} \sin 5x$$

کاربرد لاپلاس در معادلات

$$y'' + 2y' + y = 2e^{-x}$$

شرایط اولیه $\begin{cases} y(0) = 0 \\ y'(0) = 1 \end{cases}$ $\rightarrow L[y''] + 2L[y'] + L[y] = 2L[e^{-x}]$

$$p^2 L[y] - p y(0) - y'(0) + 2\left(pL[y] - y(0)\right) + L[y] = \frac{2}{p+1}$$

$$(p^2+2p+1)\,L[y] = \frac{2}{p+1} + 1$$

$$L[y] = \frac{\frac{2}{p+1}+1}{p^2+2p+1} = \frac{2}{(p+1)^3} + \frac{1}{(p+1)^2}$$

$$(p+1)^2$$

$$y = L^{-1}\left[\frac{2}{(p+1)^3} + \frac{1}{(p+1)^2}\right] = e^{-m}m^2 + e^{-m}m$$